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A SEIQR EPIDEMIC MODEL FOR THE PROPAGATION OF COVID-19 USING A FUZZY PARAMETER

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Abstract: The focus of this study is on developing an SEIQR model with fuzzy parameters for COVID-19. The SEIQR model is built around the corona virus load, efficacy of vaccine, treatment, no treatment, efficacy of face mask, and social isolation. The membership functions of the infection rate, recovery rate, and mortality rate owing to COVID-19 are employed as fuzzy parameters in the model. To get the fundamental reproduction number, the model analysis employs the matrix generation technique. The findings of the simulations reveal that the spread of COVID-19 will vary depending on the corona virus load. Similar results in reducing or preventing the spread of COVID-19 can be achieved with the use of immunization face mask, and social distancing strategy.

Keywords: Virus load, Fuzzy parameters, Basic reproduction number, Next generation matrix method

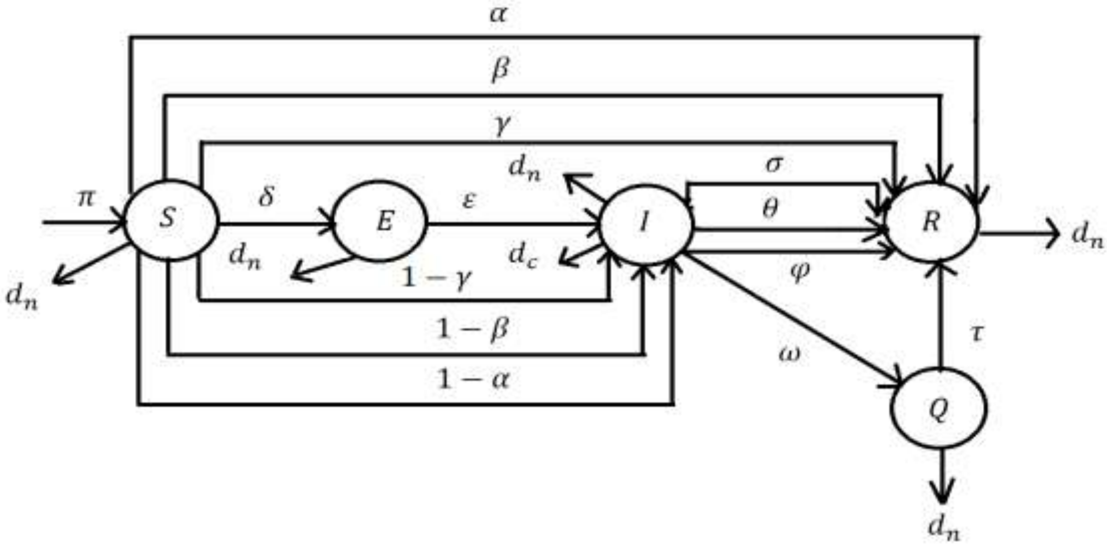
1. Introduction

The COVID-19 is a member of a broad family of lethal viruses that have infected millions of people worldwide and posed significant threats to not only individual lives but also national economies and gross domestic product. In December 2019, the first instance of respiratory illness was recorded in Wuhan, China. Since then, it has been determined to be the source of a large number of cases of pneumonia in that city. The corona virus is spread when an exposed individual breathes in infected droplets exhaled by an infected person when sneezing, breathing, or coughing. Reliable assessment of transmission dynamics is an important element of the research, along with many other problems about COVID-19 illness. Worldwide, readers of the novel COVID-19 continue to experience widespread fear. There is now a global fifth wave of this new epidemic. There are a number of strategies under consideration for stopping this fatal illness. Clinical insights, epidemiological models, and methods for preventing the spread of disease are just some of the methods proposed.

A new method for handling ecological models with fuzzy parameters in differential equations describing the dynamical system was proposed by Barros et al. (2003), Barros and Bassanezi (1997), and Bassanezi and Barros (1995). The piecewise linear treatment function is incorporated into a SIR model by Wang (2006). The optimal harvesting of food chains in a fuzzy setting was the subject of research by Sadhukhan et al. (2010). By simulating the outcomes for different parameters and analyzing the model's stability, Mishra and Prajapati (2013) established a fuzzy SEIQRS model for the spread of harmful codes in computer networks. In the context of treatment control, Mondal et al. (2015) analysed and described the dynamical behaviour of a simple SIS type epidemic model they created. Verma et al. (2018) examined an asymptotic transmission rate model of Influenza's spread, in which the transmission rate of the disease and the mortality rate were both treated as fuzzy sets. The fuzzy basic reproduction number was investigated by Verma et al. (2018) and compared to the classical model's basic reproduction number for groups of infected individuals with varying viral loads. Ullah and Khan (2020) created a mathematical model without optimum control and time-dependent control variables to investigate the spread dynamics and possible management of the COVID-19 epidemic in Pakistan. Khan and Atangana (2020) conducted in-depth research on the complex relationship between bats and unknown hosts, including humans, who serve as a potential reservoir of infection. To help people quit smoking, Allaoui et al. (2020) suggested a fuzzy model. It also contrasts the fuzzy theory's findings with those of the classical example. Irfan et al. (2021) looked into how temperature affected the spread of COVID-19 across Pakistani provinces. When building their SIR model for COVID-19, Abdy et al. (2021) opted to use fuzzy parameters. They took into account corona virus-load in addition to immunisation, therapy, and compliance with health standards. Padmapriya and Kaliyappan (2022) built a mathematical model for predicting COVID-19 using a Caputo fractional derivative with fuzzy sense. This investigation takes into account a standardized mathematical model of SEIQR with four control parameters: vaccine control, treatment control, facemask, and social distancing protocol. In this case, the metrics of infection rate, recovery rate, and mortality rate owing to COVID-19 are all considered fuzzy values that depend on the individual's virus-load.

2. Model of COVID-19 based on SEIQR:

Figure 1: The transition diagram of the model SEIQR of COVID-19 spread transmission flow



Consider a SEIQR model for Corona virus, which portrays the elements of direct transmission, including associations between the thought and tainted, the progress from being contaminated to recuperating, unadulterated birth/passing rates, immunization viability, treatment and without treatment adequacy, facial covering, social separating conventions, and passings brought about by Corona virus disease. In Fig. 1 we see a worked on portrayal of the Corona virus transmission stream, and the model is expressed numerically as follows:

$$\frac{dS}{dt} = \pi - (1-\alpha)(1-\beta)(1-\gamma)SI - (\alpha+\beta+\gamma+\delta+d_n)S \quad (1)$$

$$\frac{dE}{dt} = \delta S - (\varepsilon + d_n)E \quad (2)$$

$$\frac{dI}{dt} = (1-\alpha)(1-\beta)(1-\gamma)SI + \varepsilon E - (d_n + d_c + \sigma + \theta + \varphi + \omega)I \quad (3)$$

$$\frac{dQ}{dt} = \omega I - (\tau + d_n)Q \quad (4)$$

$$\frac{dR}{dt} = (\alpha+\beta+\gamma)S + (\sigma+\theta+\varphi)I + \tau Q - d_n R \quad (5)$$

Where S is the fraction of people who are vulnerable to infection, E is the fraction of people who are at risk of contracting the disease, I is the fraction of people who actually contract the disease, Q is the fraction of people who are quarantined, and R is the fraction of people who have recovered. Additionally, δ is the uncovered rate boundary, ε is the contamination rate boundary, θ is the recuperation rate boundary; d_n is the normal birth/demise rate boundary, α is the immunization viability boundary, σ and ω are the treatment and non treatment adequacy boundaries separately, β and γ are the viability of facemask and social removing convention, ω is the isolated rate boundary by which tainted people goes to isolated compartment, τ is the rate by which isolated people ranges to recuperated compartment and d_C is the passing rate boundary because of Corona virus. Presently, we can broaden the SEIQR model (1)- (5) by taking into account the heterogeneity of the Covid load in every person, where people with various measure of the Covid load contribute distinctively in communicating Corona virus.

3. The fuzzy SEIQR model of the COVID-19 spread-spectrum

The SEIQR model can be used to the Corona virus in (1)–(5). Let a person's Covid load equal Ψ. Since everyone's susceptibility to contamination is a part of the Covid load, we can now account for the model's heterogeneity. Therefore, the likelihood of Covid transmission during contact cooperation increases with the Covid burden Ψ in an individual. The limits ε, θ and d_C can be understood as a part of the Covid load Ψ when considering the Covid load in each individual. Models (1)–(5) can be generalized in this way to get what we'll call the "fuzzy SEIQR model," which has the following formulation:

$$\frac{dS}{dt} = \pi - (1-\alpha)(1-\beta)(1-\gamma)SI - (\alpha+\beta+\gamma+\delta+d_n)S \qquad (6)$$

$$\frac{dE}{dt} = \delta S - (\varepsilon(\Psi) + d_n)E \qquad (7)$$

$$\frac{dI}{dt} = (1-\alpha)(1-\beta)(1-\gamma)SI + \varepsilon(\Psi)E - (d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)I \qquad (8)$$

$$\frac{dQ}{dt} = \omega I - (\tau + d_n)Q \qquad (9)$$

$$\frac{dR}{dt} = (\alpha+\beta+\gamma)S + (\sigma + \theta(\Psi) + \varphi)I + \tau Q - d_n R \qquad (10)$$

Let $\varepsilon = \varepsilon(\Psi)$ be the opportunity of transmission between a thought and a tainted person with how much the Covid load Ψ. A few upsides of ε are more sensible contrasted with some others, and it transforms ε into a participation capability of fuzzy numbers. Then, the infectivity contact rate's fuzzy membership capability is described as follows:

$$\varepsilon(\Psi) = \begin{bmatrix} 0 & \Psi \le \Psi_{\min} \\ \frac{(\Psi-\Psi_{\min})(1-\alpha)(1-\beta)(1-\gamma)}{\Psi_0-\Psi_{\min}} & \Psi_{\min} < \Psi < \Psi_0 \\ (1-\alpha)(1-\beta)(1-\gamma) & \Psi_0 \le \Psi < \Psi_{\max} \end{bmatrix} \qquad (11)$$

The passing rate because of Coronavirus disease can likewise be expected as a participation capability of a fuzzy number. The capability is a rising capability of Covid load Ψ. Be that as it may, because of certain reasons, for example, an individual tainted with Coronavirus experiencing different illnesses, resistance power, accessibility of medication, and so on, the capability probably won't arrive at its most extreme worth equivalent to one. Similarly, treatment for Coronavirus will influence the passing rate because of Coronavirus disease. In this way, we accept that the most extreme worth of the capability $d_C(\Psi)$ is $(1-\sigma)(1-\varphi) + d_C$ with $0 \le \sigma \le 1$ and $0 \le \varphi \le 1$. Consequently, we can characterize the capability $d_C(\Psi)$ as follows:

$$d_C(\Psi) = \begin{bmatrix} \left((1-\varphi)-(d_C)_0\right)(1-\sigma)\frac{\Psi}{\Psi_0} + (d_C)_0 & 0 \le \Psi < \Psi_0 \\ (1-\sigma)(1-\varphi) + (d_C)_0 & \Psi_0 \le \Psi \end{bmatrix} \qquad (12)$$

Where $(d_C)_0$ $(< 0(d_C)_0 < 1)$ is the most reduced passing rate because of Coronavirus contamination and, φ are the treatment and non treatment adequacy.

The recuperation pace of the Coronavirus contamination bunch $\theta = \theta(\Psi)$ is likewise a component of the Covid load Ψ. The higher the Covid load Ψ, the more extended the recuperation interaction will take from disease. Thus, $\theta(\Psi)$ is a diminishing capability? Additionally, we can likewise think about the impact of the treatment on the pace of recuperation. Consequently the fuzzy enrollment capability can be characterized as follows:

$$\theta(\Psi) = \begin{bmatrix} (\theta_0 - 1)(1-\sigma)\frac{\Psi}{\Psi_0} + 1 & 0 \le \Psi < \Psi_0 \\ \theta_0(1-\sigma) + \sigma & \Psi_0 \le \Psi \end{bmatrix} \qquad (13)$$

where θ_0 is the least recuperation rate.

In the fuzzy SEIQR model, the enrollment capability of the disease rate $\varepsilon(\Psi)$, the recuperation rate $d_C(\Psi)$, and the passing rate $\theta(\Psi)$ because of Coronavirus contamination are treated as fuzzy boundaries of the model.

4. Points representing the state of equilibrium:

There are two harmony focuses in model (6)–(10), to be specific the illness free balance point and the endemic balance point. To decide these two harmony focuses, every one of the situations in the situations should be equivalent to nothing, that is $\frac{dS}{dt} = 0, \frac{dE}{dt} = 0, \frac{dI}{dt} = 0, \frac{dQ}{dt} = 0, \frac{dR}{dt} = 0$

$$\pi - (1-\alpha)(1-\beta)(1-\gamma)SI - (\alpha+\beta+\gamma+\delta+d_n)S = 0 \qquad (14)$$

$$\delta S - (\varepsilon(\Psi) + d_n)E = 0 \qquad (15)$$

$$(1-\alpha)(1-\beta)(1-\gamma)SI + \varepsilon(\Psi)E - (d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)I = 0 \qquad (16)$$

$$\omega I - (\tau + d_n)Q = 0 \qquad (17)$$

$$(\alpha+\beta+\gamma)S + (\sigma + \theta(\Psi) + \varphi)I + \tau Q - d_n R = 0 \qquad (18)$$

Then, at that point, the equilibrium focuses for S, E, I, Q, R are as per the following.

5. The point of equilibrium where there is no sickness according to the SEIQR fuzzy model

The points of equilibrium for infection free are conditions where there is no spread of Coronavirus, in particular $E = E^0 = 0$ and $I = I^0 = 0$. Consequently, from Conditions (14-18), we get

$\pi - (\alpha + \beta + \gamma + \delta + d_n)S = 0$

$S = S^0 = \frac{\pi}{\alpha+\beta+\gamma+\delta+d_n}$

$E = E^0 = 0$

$I = I^0 = 0$

$Q = Q^0 = 0$

$R = R^0 = \frac{(\alpha+\beta+\gamma)S}{d_n} = \frac{(\alpha+\beta+\gamma)\pi}{d_n(\alpha+\beta+\gamma+\delta+d_n)}$

Consequently, the infection free harmony point for the SIR fuzzy model (6)- (10) is

Thus, the disease-free equilibrium point for the SEIQR fuzzy model (6)–(10) is

$E^0 = \left(\frac{\pi}{\alpha+\beta+\gamma+\delta+d_n}, 0, 0, 0, \frac{(\alpha+\beta+\gamma)\pi}{d_n(\alpha+\beta+\gamma+\delta+d_n)}\right)$

6. Indicators of endemic balance equilibrium:

$S = S^* \neq 0$, $E = E^* \neq 0, I = I^* \neq 0, Q = Q^* \neq 0, R = R^* \neq 0$ are all examples of endemic balancing foci, all of which have the potential to transmit disease. Therefore, the following endemic balancing foci for the SEIQR fuzzy model are derived from Eqs. (6)-(10):

$$\pi - (1-\alpha)(1-\beta)(1-\gamma)SI - (\alpha+\beta+\gamma+\delta+d_n)S = 0 \quad (19)$$

$$\delta S - (\varepsilon(\Psi) + d_n)E = 0 \quad (20)$$

$$(1-\alpha)(1-\beta)(1-\gamma)SI + \varepsilon(\Psi)E - (d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)I = 0 \quad (21)$$

$$\omega I - (\tau + d_n)Q = 0$$

$$(\alpha+\beta+\gamma)S + (\sigma + \theta(\Psi) + \varphi)I + \tau Q - d_n R = 0 \quad (22)$$

Addressing conditions (19), (20) and (21), we get quadratic condition in I

$$I^2[(1-\alpha)(1-\beta)(1-\gamma) + (d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)(\varepsilon(\Psi) + d_n)] - I[(1-\alpha)(1-\beta)(1-\gamma)\pi(\varepsilon(\Psi) + d_n) + (\alpha+\beta+\gamma+\delta+d_n)(d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)(\varepsilon(\Psi) + d_n)] + [\pi\{(\alpha+\beta+\gamma+\delta+d_n)(\varepsilon(\Psi)+d_n) - \delta\varepsilon(\Psi)\} - (\alpha+\beta+\gamma+\delta+d_n)\pi(\varepsilon(\Psi)+d_n)] = 0 \quad (23)$$

Let $I = I^* = f(\Psi)$ is the arrangement of condition (23)

In light of the arrangement of condition (23), we get

$$S^* = \frac{[\pi(\varepsilon(\Psi)+d_n) - (d_n + d_c(\Psi) + \sigma + \theta(\Psi) + \varphi + \omega)(\varepsilon(\Psi)+d_n)f(\Psi)\ ]}{[(\alpha+\beta+\gamma+\delta+d_n)(\varepsilon(\Psi)+d_n) - \delta\varepsilon(\Psi)]} \quad (24)$$

$$E^* = \frac{\delta}{(\varepsilon(\Psi)+d_n)} S^* \quad (25)$$

$$Q^* = \frac{\omega}{\tau + d_n} I^* \quad (26)$$

$$R^* = \frac{(\alpha+\beta+\gamma)S^* + (\sigma+\theta(\Psi)+\varphi)I^* + \tau Q^*}{d_n} \quad (27)$$

7. The SEIQR Fuzzy Model's Basic Reproduction Number

The basic reproductive number R_0 for framework (1)- (5) is resolved utilizing the future network strategy. In light of Conditions. (1)- (5), to decide R_0:

$$\mathscr{F} = \begin{bmatrix} \delta \\ (1-\alpha)(1-\beta)(1-\gamma)I \\ 0 \end{bmatrix}, \mathscr{V} = \begin{bmatrix} (\varepsilon + d_n)E \\ (d_n + d_c + \sigma + \theta + \varphi + \omega)I - \varepsilon E \\ (\tau + d_n)Q - \omega I \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (1-\alpha)(1-\beta)(1-\gamma) & 0 \\ 0 & 0 & 0 \end{bmatrix}, V = \begin{bmatrix} (\varepsilon + d_n) & 0 & 0 \\ -\varepsilon & (d_n + d_c + \sigma + \theta + \varphi + \omega) & 0 \\ 0 & -\omega & (\tau + d_n) \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\varepsilon(1-\alpha)(1-\beta)(1-\gamma)}{(\varepsilon+d_n)(d_n+d_c+\sigma+\theta+\varphi+\omega)} & \frac{(1-\alpha)(1-\beta)(1-\gamma)}{d_n+d_c+\sigma+\theta+\varphi+\omega} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

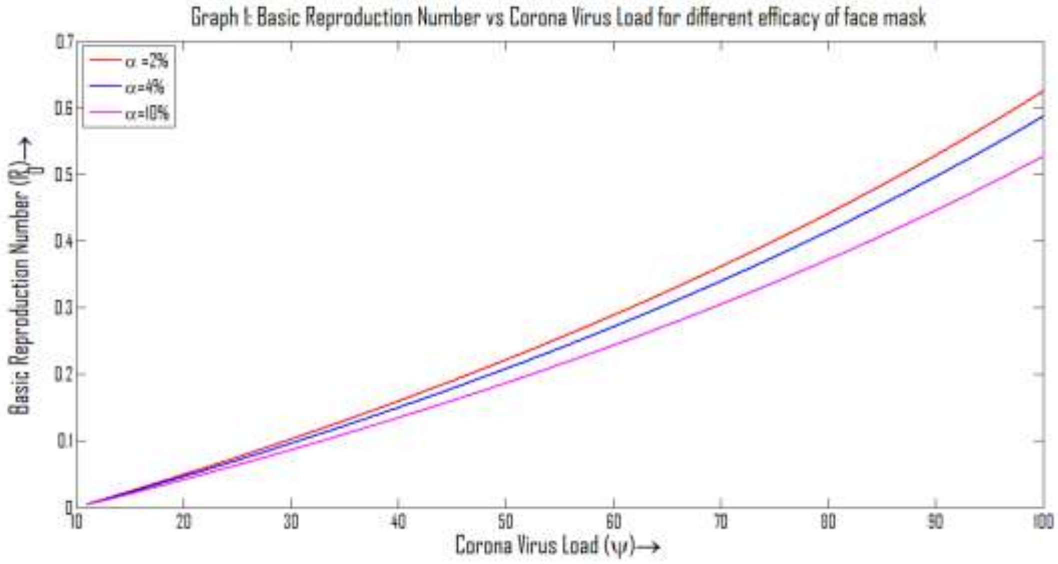
The biggest Eigen value of FV^{-1} is the Basic Reproduction Number (R_0), or

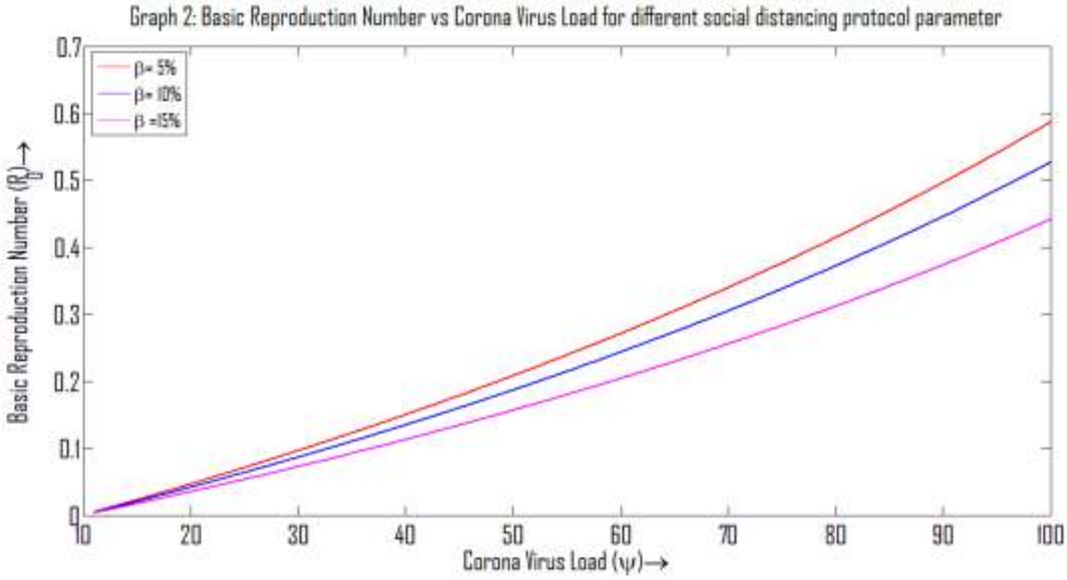
$$R_0 = \frac{\varepsilon(1-\alpha)(1-\beta)(1-\gamma)}{d_n+d_c+\sigma+\theta+\varphi+\omega}$$

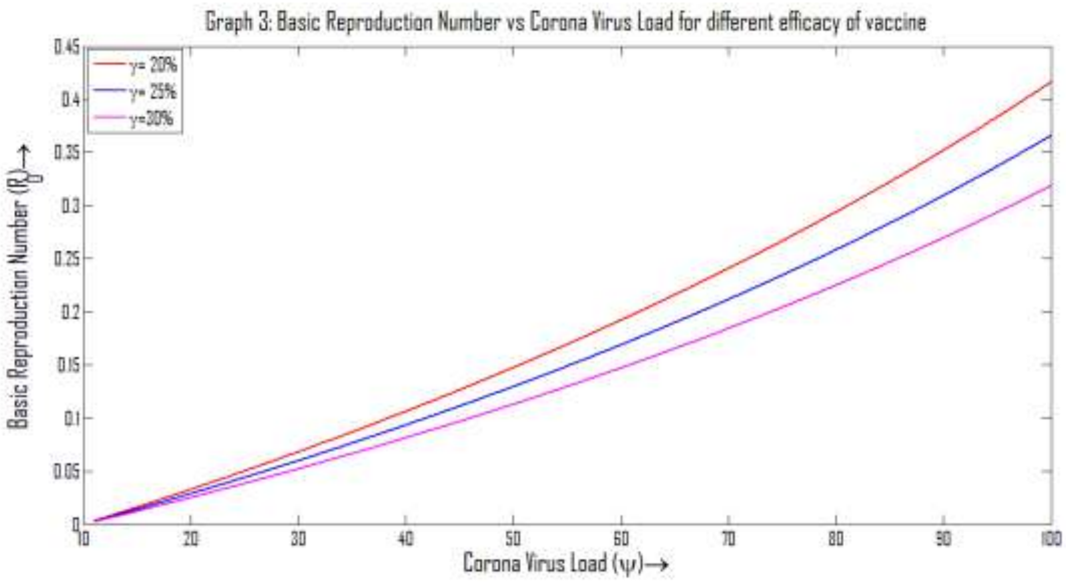
The Basic reproduction number of SEIQR fuzzy model is

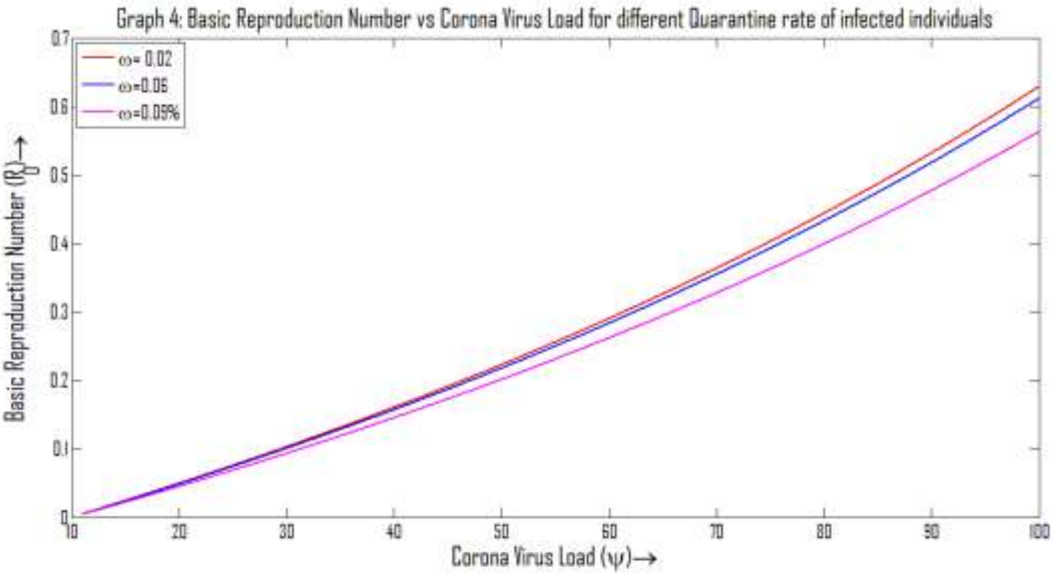
$$R_0^{fuzzy} = \frac{\varepsilon(\Psi)(1-\alpha)(1-\beta)(1-\gamma)}{d_n+d_c(\Psi)+\sigma+\theta(\Psi)+\varphi+\omega}$$

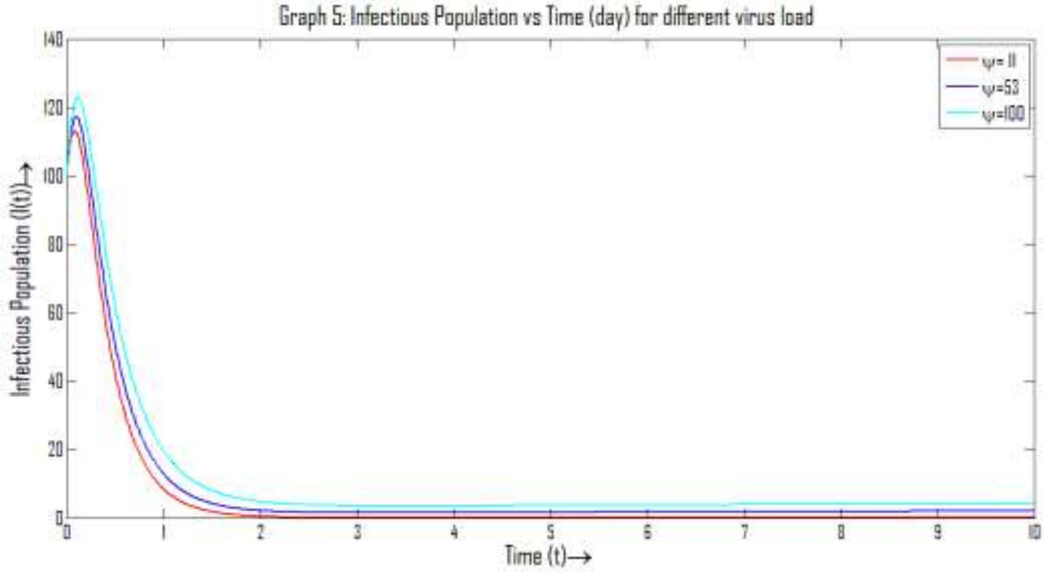
8. Numerical Simulation and Results-

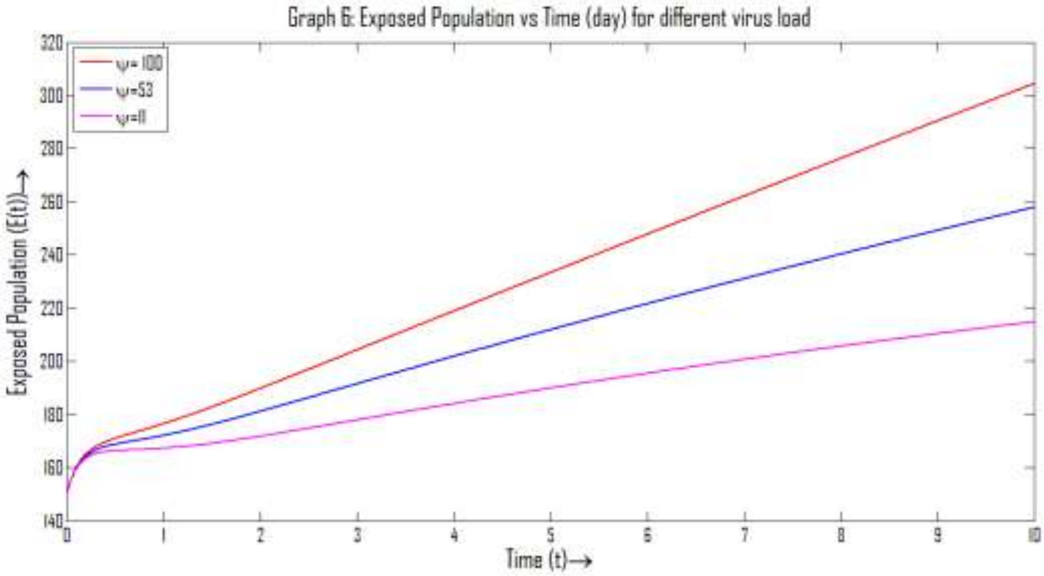


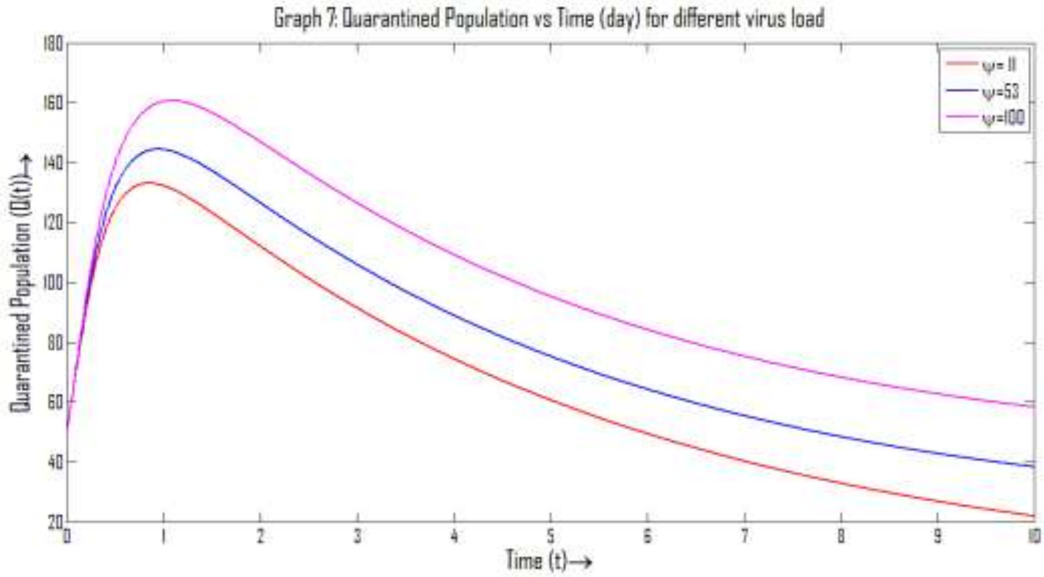


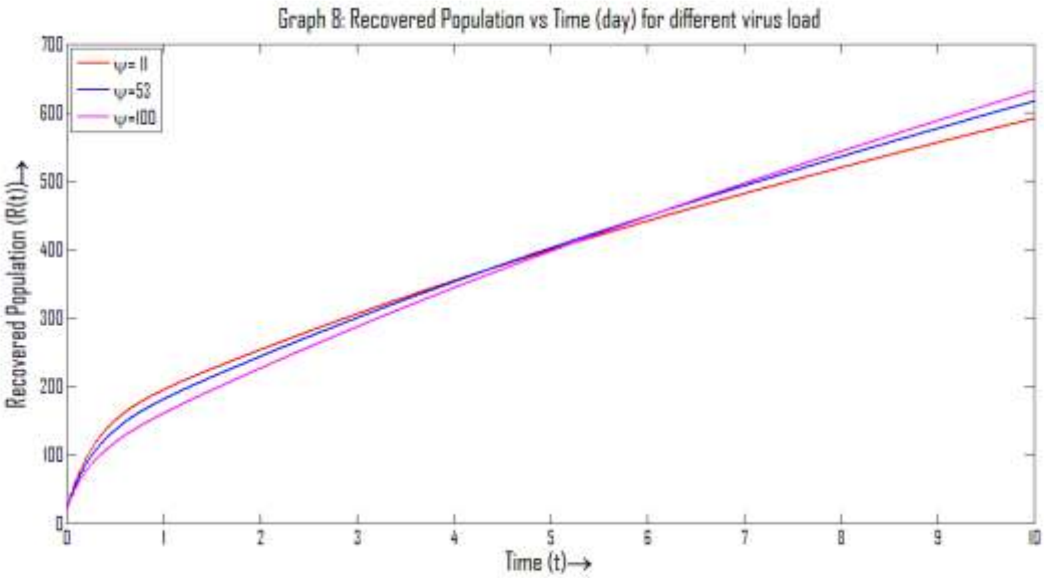


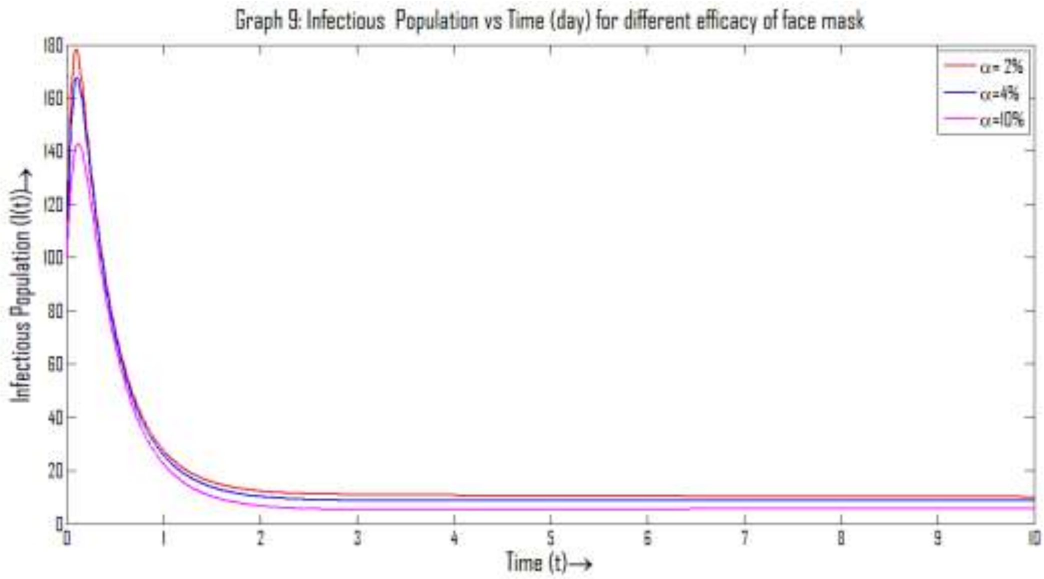

Graph 4: Basic Reproduction Number vs Corona Virus Load for different Quarantine rate of infected individuals
ω= 0.02
ω=0.06
ω=0.09%
Basic Reproduction Number (R0)→
Corona Virus Load (ψ)→

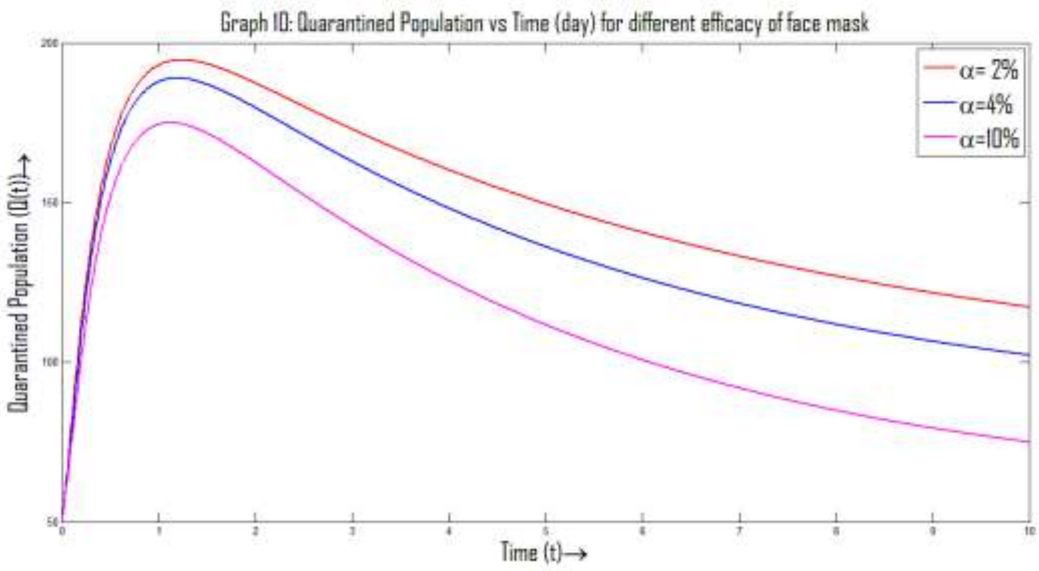

Graph 5: Infectious Population vs Time (day) for different virus load
ψ= 11
ψ=53
ψ=100
Infectious Population (I(t))→
Time (t)→


Graph 6: Exposed Population vs Time (day) for different virus load
ψ= 100
ψ=53
ψ=11
Exposed Population (E(t))→
Time (t)→


Graph 7: Quarantined Population vs Time (day) for different virus load
Quarantined Population (Q(t))→
Time (t)→
ψ= 11
ψ=53
ψ=100
0
1
2
3
4
5
6
7
8
9
10
20
40
60
80
100
120
140
160
180


Graph 8: Recovered Population vs Time (day) for different virus load
Recovered Population (R(t))→
Time (t)→
ψ= 11
ψ=53
ψ=100
0
1
2
3
4
5
6
7
8
9
10
0
100
200
300
400
500
600
700


Graph 9: Infectious Population vs Time (day) for different efficacy of face mask
Infectious Population (I(t))→
Time (t)→
α= 2%
α=4%
α=10%
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
140
160
180



In order to perform numerical simulations, the initial values for S, E, I, Q, R are $S(0) = 160, E(0) = 150, I(0) = 100, Q(0) = 50, R(0) = 20$

The upsides of Ψ_0 , Ψ_{min} and Ψ ought not entirely set in stone by a virologist, while the worth of parameter φ ought still up in the air by a doctor. For every one of the controlled parameters, to be specific the boundaries of adequacy of vaccine, treatment, without treatment, viability of facial covering, social separating and the Covid load. Reproductions are completed multiple times for some of the parameters with various qualities. From graphs (1)-(3), it tends to be made sense of that assuming that a viability of facial covering, social removing convention parameter and viability of immunization have not been executed, the fundamental propagation number increments separately, and that implies that an individual with Corona virus can taint others quickly. Additionally, from graph (4), it is seen that in the event that quarantine rate of irresistible populace increments, fundamental generation number of Corona virus diminishes and pandemic will in general lessen.

The reenactment results for the Covid load are introduced in graphs (5)- (8). In light of these graphs, the Covid load is exceptionally high, the Corona virus flare-up won't ever evaporate in the populace. In the mean time, if the infection crown load is diminishes, the pandemic will in general diminish, and the Corona virus flare-up will vanish in the populace.

The reproduction after effects of adequacy of facial covering on irresistible and isolated populaces are introduced in graphs (8) and (9). In view of these figures, on the off chance that the viability of facial covering is just 2%, Corona virus will become endemic in the populace. Be that as it may, in the event that acquiescence to follow wellbeing conventions is more than 10 %, the Corona virus flare-up will diminish in the populace. From the reenactment after effects of adequacy of vaccine it very well may be seen that, for similar qualities of γ, the outcomes will be something similar. This shows that adequacy of facial covering and the immunization meaningfully affect the spread of Corona virus.

9. Conclusion

The spread of COVID-19 has been studied using an SEIQR model, with vaccination, therapy, health protocol application, and corona virus-load all taken into account. In this research, the parameters ε, θ and d_C are represented as fuzzy parameters since they are interpreted as membership functions of fuzzy numbers. Vaccination, facemasks, and a social distancing regimen all show promise in the simulations for reducing the spread of COVID-19. Similarly, treatment can delay or halt the spread of COVID-19, but not nearly as effectively as the vaccination, facemask, and social isolation approach.

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